

# Complex poles and oscillatory screened potential in QED plasmas

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We discuss the mechanism for the oscillatory behavior of the static interparticle potential in a degenerate electron plasma. This behavior, observed in metallic alloys, is commonly referred to as 'Friedel oscillations', and its origin associated to the Kohn singularity. We show that, although this interpretation is adequate for large distances, the oscillations at short distances originate from a complex pole of the in-medium photon propagator in the complex  $q$ -plane, which exists aside the (purely imaginary) Debye pole. Such short-range oscillations can be physically discriminated if they remain at temperatures for which Friedel oscillations have already disappeared. This is suggested by finite temperature calculations in non-electromagnetic plasma models showing a similar pole structure.

It is well-known that wave propagation and particle interaction in vacuum are substantially modified by medium effects inside a plasma at a given density and temperature [1–3]. In the case of a QED plasma, ordinary transverse photons (with respect to the wavelength vector) acquire an 'effective mass', and are governed by a dispersion relation which substantially differs from the vacuum wave equation. In addition to these photons, there appear also longitudinal modes with different propagation properties [4,5]. These modes arise from the poles of the photon propagator in the medium, and give raise to a number of phenomena in plasma and solid state physics. It is then important to analyze the structure of the photon propagator and its physical consequences. In this letter, we will concentrate on this study for the static photon propagator at zero temperature in the complex  $q$ -plane (where  $q$  is the momentum of the photon), and examine some of the consequences of our findings when considering the interparticle potential in the plasma.

We calculate the quasi-photon propagator in the RPA approximation to the dielectric function, and include a local field correction, as given by Ichimaru [6] (we have also performed calculations with other parameterizations of this correction). The expression of the static potential between two point-like impurities of charge  $e$  is obtained from this propagator, and reads :

$$V(r) = \frac{e^2}{4\pi^2 r} \text{Im} \int_{-\infty}^{\infty} dq \frac{q e^{iqr}}{D(q)} \quad (1)$$

where  $D(q) = q^2 \epsilon(\omega = 0, q)$ ,  $\epsilon(\omega, q)$  is the dielectric function and  $r$  the interparticle distance.

The denominator  $D(q)$  contains the function  $f(q) = \log \left| \frac{q+2p_f}{q-2p_f} \right|$ , with  $p_f$  the Fermi momentum of the electrons. This function can be written, for real values of  $q$ , as follows :

$$f(q) = \begin{cases} f_1(q) \equiv 2 \arg \tanh\left(\frac{q}{2p_f}\right), & \left|\frac{q}{2p_f}\right| < 1 \\ f_2(q) \equiv 2 \arg \tanh\left(\frac{2p_f}{q}\right), & \left|\frac{2p_f}{q}\right| < 1 \end{cases} \quad (2)$$

We define the functions  $D_1(q)$  and  $D_2(q)$ , which are obtained by replacing  $f(q) \rightarrow f_1(q)$  and  $f(q) \rightarrow f_2(q)$ , respectively, in  $D(q)$ . Then, Eq. (1) becomes

$$V(r) = \frac{e^2}{4\pi^2 r} \text{Im} \left[ \int_{-\infty}^{-2p_f} dq \frac{q e^{iqr}}{D_2(q)} + \int_{2p_f}^{\infty} dq \frac{q e^{iqr}}{D_2(q)} + \int_{-2p_f}^{2p_f} dq \frac{q e^{iqr}}{D_1(q)} \right] \quad (3)$$

We next consider the analytical continuation of the function  $D_1(q)$  to the complex  $q$ -plane, and apply the residue theorem in order to transform the last integration in Eq. (3) into the integration domain of the first two integrals in

the same equation. After this operation, and with the help of the symmetry properties of the photon propagator due to the Onsager relations, Eq. (1) can be cast under the form :

$$V(r) = V_P(r) + V_C(r) \quad (4)$$

Here,  $V_P(r)$  is the contribution of the residues of  $1/D_1(q)$  in the region  $\text{Im}(q) > 0$ . We have also defined

$$V_C(r) = \frac{e^2}{2\pi^2 r} \text{Im} \left[ \int_{2p_f}^{\infty} dq q e^{iqr} \left( \frac{1}{D[f_2(q)]} - \frac{1}{D[f_2(q) + i\pi]} \right) \right] \quad (5)$$

The analytical structure of  $1/D_1(q)$  on the upper half-plane is shown in Fig. 1, where we have also drawn the integration contour used to perform the integration. As we discuss later, in addition to the well-known 'Debye pole', lying on the imaginary axis, *there exist complex poles*, which are responsible for strong oscillations of the interparticle potential at short distances.

In fact, while the oscillatory behavior of the screened potential created by a ionic impurity in metallic alloys is known since the works of Friedel [7] (see also [1,8]), its algebraic decay with distance has been associated to the existence of the Kohn singularity [9] in the photon self-energy, induced by the sharp edge of the degenerate electron distribution: when considering energy and momentum conservation in the collisions between the electrons and soft quasi-photons, we can easily see that only a fraction of the electrons can interact with quasi-photons and, therefore, screening of a Fermi gas at  $T=0$  K is less effective than screening at finite temperature. At the threshold of the interaction, i. e. at the threshold of electron-hole creation, the momentum of the quasi-photon is equal to the diameter of the Fermi sphere. It must be noticed that the photon self-energy presents a singularity at this point. The presence of a singularity permits to obtain, with the help of Lighthill's method, the asymptotic form ( $r \rightarrow \infty$ ) of the potential, as an expansion in terms of the form  $\cos(2p_f r)$  and  $\sin(2p_f r)$ , damped as negative powers of  $r$ , and enhanced by powers of  $\log(4p_f r)$  [10]. One has to remember, for further discussion, that Lighthill's method consists essentially in replacing the Fourier (anti) transform of a non-analytical function by the (anti) transform of the (formal) expansion of that function around its singular points.

In Fig. 2 we have plotted the result for the first, dominant term ( $\sim \cos(2p_f r)/r^2$ ), of Lighthill's expansion (dotted line) for the potential around an impurity placed into an electron gas with <sup>1</sup>  $r_s = 3$ , where  $r_s$  is the characteristic interparticle distance in units of the Bohr's radius. In the same figure we have also plotted for comparison (solid line) the result of a numerical calculation of  $V(r)$ , as obtained by direct integration from Eq. (1). As we see, the Lighthill expansion converges to the true result for large values of  $r$ . An even better approximation to the exact result is given by  $V_C(r)$ , as defined above, and represented by the dashed line. The reason for this good agreement is that  $V_P(r)$  exponentially goes to zero when  $p_f r \gg 1$ , and thus only  $V_C(r)$  is important for long distances. Of course, the advantage of the Lighthill's method for large distances is to provide a systematic way to construct corrections to the simple formula we used here, and to give an analytical expression for these terms, although they become rapidly cumbersome as the order of the approximation increases.

However, as we go to shorter distances, the situation becomes quite different. The reason is that, in this range, the dominant contribution to the form of the potential is given by the poles of the photon propagator in the complex plane, which induce exponentially-damped oscillations. This is clearly seen in Fig. 3, where we have represented the different contributions to the potential for the same value  $r_s = 3$ , but for a shorter distance range. As before, the exact result (obtained from direct numerical integration) has also been plotted. In order to calculate  $V_P(r)$ , we have to locate numerically the zeros of  $D_1(q)$  in the upper half-plane, while  $V_C(r)$  is calculated by numerical integration. As we already mentioned, we find a zero lying on the imaginary- $q$  axis, which is commonly referred to as the 'Debye pole'. In addition to this, there exists a genuinely complex pole, with both a real and an imaginary part, which gives raise to exponentially-damped oscillations. As we can see, the contribution from the poles  $V_P(r)$ , represented by the dashed-dotted line, gives a much closer result to the exact line than the Lighthill term. We have also plotted, for the sake of completeness, the contribution from  $V_C(r)$ , which accounts for the difference between the poles term and the exact potential<sup>2</sup>. Thus, for this range of distances, the potential is dominated by the pole contribution. As a matter of fact, for the present value of  $r_s$  the contribution of the Debye and complex poles to  $V_P(r)$  are of the same order.

As the density is changed, the relative importance of these two contributions to  $V_P(r)$  changes. This can be understood from Fig. 4, where we have shown the evolution of the real and imaginary parts of the complex pole, and the imaginary part of the Debye pole. For large densities ( $r_s \lesssim 2$ ),  $V_P(r)$  reduces to the well-known Debye screened

<sup>1</sup>This is a typical value corresponding to metals.

<sup>2</sup> We have verified that the addition of  $V_C(r)$  to  $V_P(r)$  reproduces the exact potential, within machine-size precision numbers.

potential, since the contribution arising from the complex pole is strongly damped. For larger values of  $r_s$ , however, the situation is reversed, since  $V_P(r)$  is dominated by the complex pole and, therefore, this term alone constitutes a good approximation to the exact potential.

Let us summarize our results. We have studied the static interparticle potential in a QED electron plasma at zero temperature. This can be obtained from the knowledge of the dielectric function (or equivalently, of the photon polarization), which was taken from the RPA approximation, with local field corrections included. For the latter, we adopted the formulae given by Ichimaru, although we have performed also calculations with the formulae of Ref. [13], with very similar results. We found that the potential shows an oscillatory behavior for the whole distance range. This behavior has been traditionally referred to as Friedel oscillations, and its origin attributed to the Kohn singularity. We have shown that one can get a much better insight into this behavior through the study of the analytic structure of the photon propagator in the complex- $q$  plane. We find that, in addition to the well-known Debye pole on the imaginary axis, there exists a complex pole, which accounts for the oscillations of the potential at short ( $r \gtrsim 1\text{\AA}$ ) distances.

A similar pole structure, and the associated short-range oscillations, has been found in other (non electromagnetic) plasma models describing the one-pion exchange [11] and the exchange of other mesons in nuclear matter [12]. In these cases, one can easily show that the separation of the screened potential into a short-range component and a Friedel-like component is not merely a mathematical artifice, but in fact can have physically observable consequences. Indeed, the phenomenon we have revisited (Friedel oscillations) and the new one obtained here come from two different effects. Friedel oscillations arise from the non-analytical behavior of the Fourier transform of the potential created by an impurity, while our results come from the zeros of the dispersion relation, in the complex plane, of the quasi-photon momentum. Using Maxwell equations, it can be easily obtained the charge distribution from the potential of the system. It comes out that the form of the charge and potential distribution are qualitatively identical. The presence of an impurity, then, leads to the apparition of a spatial static structure of charges in the medium, which is characteristic of a highly-interacting system. As long as the temperature increases, the kinetic energy of particles increases, and this structure breaks down. At the same time, the non-analytic behavior of the photon self-energy disappears at finite temperature, and therefore Friedel oscillations are exponentially damped with temperature.

This effect has been explicitly shown in [11] for the one-pion exchange. We also found that the complex pole is more stable, and remains present (as well as the associated short-range oscillations), for higher temperatures. This fact should permit to experimentally distinguish between both phenomena.

Even if the immediate extrapolation of these results to the present case might not be justified, the similarity of the analytic structure of the dressed boson propagators in both systems at  $T = 0$  suggest that a similar behavior can be expected for the electronic plasma. If such a thermal behavior is genuine, it could probably be experimentally tested in heated liquid metals, although a calculation of the characteristic temperatures at which Friedel and short-range oscillations disappear will be necessary.

Consequently, in order to get a better understanding of this phenomena and their possible consequences in plasma physics, it would be interesting to perform a study similar to the present one at non-zero temperature. Unfortunately, to do this we need a local-field correction formula valid at finite temperature. To our knowledge, such formula has not been given yet in the literature.

### Acknowledgments

This work has been partially supported by the Spanish Grants DGES PB97-1432 and AEN99-0692.

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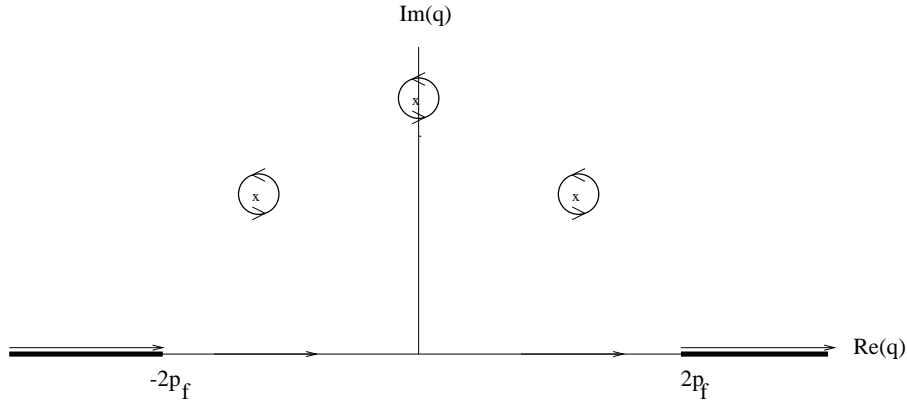


FIG. 1. Analytical structure of the function  $1/D_1(q)$  in the complex  $q$ - plane. Crosses indicate poles (discussed in the text). Thick lines correspond to branch cuts. The integration contour is shown by thin lines with arrows.

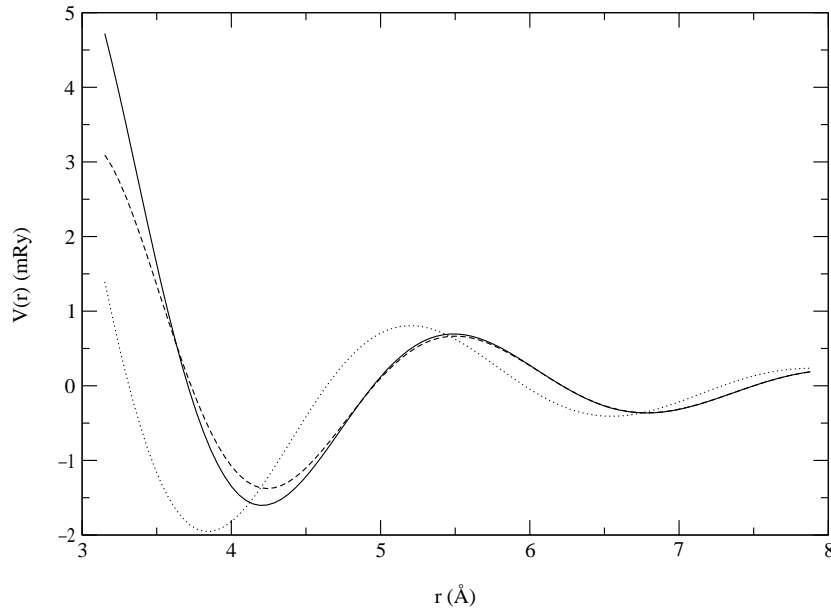


FIG. 2. Two different approximations to the interparticle potential at large distances. The dotted line shows the first term of the Lighthill expansion, and the dashed line corresponds to  $V_C(r)$ . Also shown for comparison is the exact result obtained by direct numerical calculation (solid line).

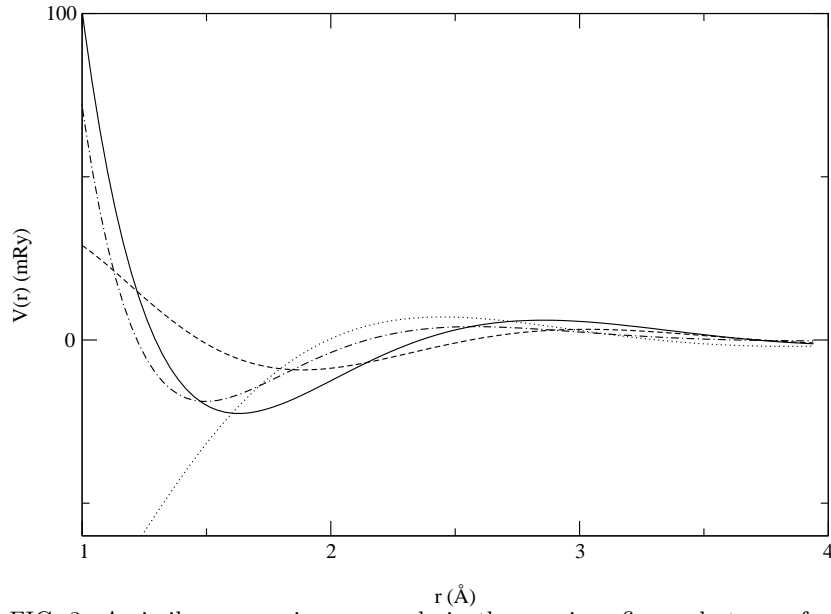


FIG. 3. A similar comparison as made in the previous figure, but now for shorter distances. We have included here the crucial contribution  $V_P(r)$  from the poles (dashed-dotted line).

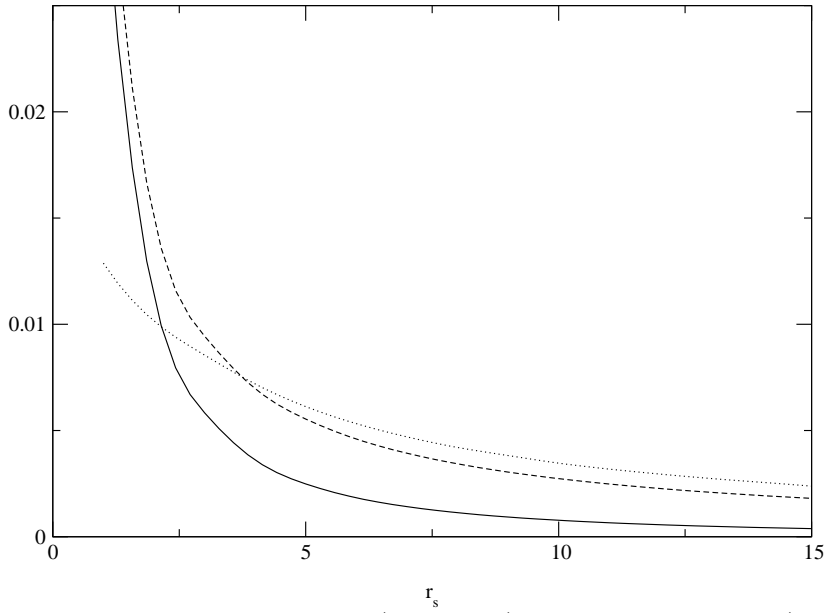


FIG. 4. Evolution of the real part (dashed line) and the imaginary part (solid line) of the complex pole as the parameter  $r_s$  changes. The dotted line shows the purely imaginary Debye pole. All magnitudes in ordinates are expressed in units of the electron mass.